static pressure and temperature at altitudes between 90,000 and 220,000 ft. Operation has been demonstrated at pressures up to 1250 psia, running times up to 2 min, and input powers up to 10.2 Mw (at over 60% efficiency for the higher mass flows). Air weight flows up to 5.0 lb-sec$^{-1}$ have been run. Units have also been employed for materials testing and in supersonic combustion experiments. The propulsion tunnel utilizes interchangeable throats to produce Mach 7 and 10 flows, the corresponding design altitudes being 110,000 and 155,000 ft. Good agreement between measured and calculated boundary layers has been demonstrated.

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# Gridding of Satellite Observations


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**Gridding is the association of earth coordinates with satellite observations. This paper describes a geometrical method for gridding the pictorial frames of cloud data obtained by the vidicon system in the Nimbus Satellite. It is shown that the particular formulation of the problem leads to an explicit bilateral mapping procedure between earth coordinates and image plane coordinates that has desirable computational consequences. Results of a typical gridding computation are presented.**


## Nomenclature

$X, Y, Z$ = coordinates of geocentric inertial axis system

$D$ = distance from origin of geocentric inertial axis system to satellite

$R, P, W$ = local coordinate system on the satellite orbit, with $R$ and $W$ axes in orbit plane

$X', Y', Z'$ = body axes; origin is coincident with origin of the $R, P, W$ axes

$\bar{X}, \bar{Y}$ = normalized variables that range between $-1$ and $+1$, in traversing the camera image planes

$e$ = subscripted, denotes unit vector along specified coordinate axis

$P_{\bar{X}}, P_{\bar{Y}}$ = scanning planes that sweep the cameras, specified by a line of constant $\bar{X}$ or $\bar{Y}$ and a point at center of camera lens

$U_{\bar{X}}, U_{\bar{Y}}$ = unit normals to the $P_{\bar{X}}, P_{\bar{Y}}$ planes, respectively

$\phi, \theta$ = surface coordinates (latitude and longitude, respectively) on nonrotating earth model

$\phi', \theta'$ = latitude and longitude, respectively, on rotating earth model

$\lambda, \mu, \nu$ = direction cosines relative to inertial axes

$\lambda', \mu', \nu'$ = direction cosines relative to body axes

$\Omega$ = longitude of ascending node, deg

$i$ = angle of inclination, deg

$\theta_R, \theta_P, \theta_W$ = angles of roll, pitch, and yaw

$\omega_e$ = earth's angular velocity


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## Introduction

A MAJOR problem inherent in the use of satellites for observation purposes is the difficulty of "gridding" an observed event, i.e., relating it to a geographic point on the earth. To know that a cyclonic formation exists is certainly of less value than knowing that a cyclonic formation is present in the Texas panhandle. Pinpointing observations geographically also affords a dynamic interpretation of events, and data which provide knowledge of the motion of cloud formations are of significant value for weather prediction.

Two gridding techniques that have merit are: 1) relation of observed events to known landmarks on the earth's surface, and 2) use of data on the orbital and attitude position of the vehicle to infer the associated latitude and longitude of an observation by geometrical analysis. The first method is simple and most accurate, but it has the drawback that landmarks are often obscured by darkness or clouds, and therefore we cannot rely on it alone. The second (geometric) method, though inherently less accurate, can always provide earth coordinates for observed data. This paper develops the latter approach for the gridding of photographic cloud data obtained by the Nimbus Meteorological Satellite, which is an attitude-stabilized, earth-oriented vehicle with three vidicon cameras. The left, center, and right cameras are disposed to cover a wide lateral sweep of the earth by a succession of snapshots along the orbit.[1]

A method is presented by which the $\bar{X}$, $\bar{Y}$ coordinate values in the image plane of a vidicon camera can be associated with latitude $\phi'$ and longitude $\theta'$ values on the surface of the earth. The method proceeds from a consideration (relative to an inertial coordinate system) of an idealized oblate spheroid earth model as it is cut by an intersecting plane (Fig. 1). The plane is specified in normal form by giving a point $(X_p, Y_p, Z_p)$ in the plane, and the direction cosines of a vector perpendicular to the plane. Points in the image planes of the cameras are suitably defined by a set of scanning planes.

Figure 2 depicts the image plane of the center camera as it is scanned by two imaginary planes $P_{\bar{X}}$ and $P_{\bar{Y}}$, which are defined by lines of constant $\bar{X}$ or $\bar{Y}$, and a point at the center of the camera lens. If one considers a fixed body system of axes $X'$, $Y'$, $Z'$, then general expressions can be readily determined for the direction cosines of the scanning planes relative to these axes. The left and right cameras could be represented as $+35°$ and $-35°$ rotations, respectively, of Fig. 2 around the $X'$ axis. Through a system of transformations from body axes to local (roll, pitch, and yaw) axes on the orbit, and from local axes to the inertial axis system, one can define the manner in which the camera scanning planes are represented in inertial space. The local roll, pitch, and yaw axes are related to inertial axes by three parameters: $\Omega$, the longitude of ascending node; $i$, the angle of inclination; and the true anomaly. With an ability to represent the camera scan planes in the inertial space, equations for the earth surface curves can be written.

## Scanning Planes and Transformations

Unit normals to the $P_{\bar{X}}$ and $P_{\bar{Y}}$ planes can be expressed in terms of unit vectors along the body axes:

$$U_{\bar{X}} = \lambda'(\bar{X})e_{X'} + \mu'(\bar{X})e_{Y'} + \nu'(\bar{X})e_{Z'}$$
$$U_{\bar{Y}} = \lambda'(\bar{Y})e_{X}, + \mu'(Y)e_{Y'} + \nu'(Y)e_{Z'}$$

The *direction cosines* for the scanning plane $P_{\bar{X}}$ of the center camera (Fig. 2) are:

$$\begin{aligned} \lambda_0'(\bar{X}) &= [1 + (\bar{X} \tan 18.5°)^2]^{-1/2} \\ \mu_0'(\bar{X}) &= 0 \\ \nu_0'(\bar{X}) &= \bar{X} \tan 18.5° \cdot [1 + (\bar{X} \tan 18.5°)^2]^{-1/2} \end{aligned} \tag{1}$$

and for the scanning plane $P_{\bar{Y}}$:

$$\begin{aligned} \lambda_0'(Y) &= 0 \\ \mu_0'(Y) &= [1 + (\bar{Y} \tan 18.5°)^2]^{-1/2} \\ \nu_0'(Y) &= \bar{Y} \tan 18.5° \cdot [1 + (Y \tan 18.5°)^2]^{-1/2} \end{aligned} \tag{1'}$$

The body and local coordinate systems have been chosen such that for zero roll, pitch, and yaw angles, the $X'$ axis is aligned with the roll axis $R$, the $Y'$ axis is coincident with the pitch axis $P$, and the $Z'$ axis is coincident with the yaw axis $W$. These two sets of axes are related through a set of Euler-angle-type transformations. It is to be noted that for large angles of rotation, the order of application of the transformations is significant; however, for small angular displacements, the transformations can be combined into an order independent matrix that relates the body system to the local system. Thus,

$$\begin{bmatrix} e_{X'} \\ e_{Y'} \\ e_{Z'} \end{bmatrix} = \begin{bmatrix} 1 & \theta_W & -\theta_P \\ -\theta_W & 1 & \theta_R \\ \theta_P & -\theta_R & 1 \end{bmatrix} \begin{bmatrix} e_R \\ e_P \\ e_W \end{bmatrix} \tag{2}$$

The transformation relating a vector in the local coordinate system to a vector represented in the inertial system is obtained as follows. The yaw axis $W$ is always directed toward the center of the earth. The roll axis $R$ is aligned with the direction of motion of the satellite. In conjunction with the roll and yaw axes, the pitch axis forms a right-handed orthogonal Cartesian system. The direction cosines of the normal to the orbit plane relative to the inertial $(X, Y, Z)$ axes are (see Fig. 1):

$$\begin{aligned} \lambda_{OP} &= \sin i \sin\Omega \\ \mu_{OP} &= -\sin i \cos\Omega \\ \nu_{OP} &= \cos i \end{aligned} \tag{3}$$

from which one can readily determine the unit vector aligned

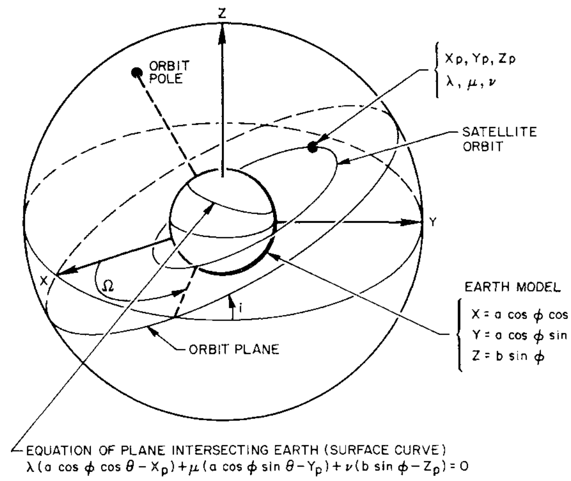


**Fig. 1 Earth-satellite geometry.**

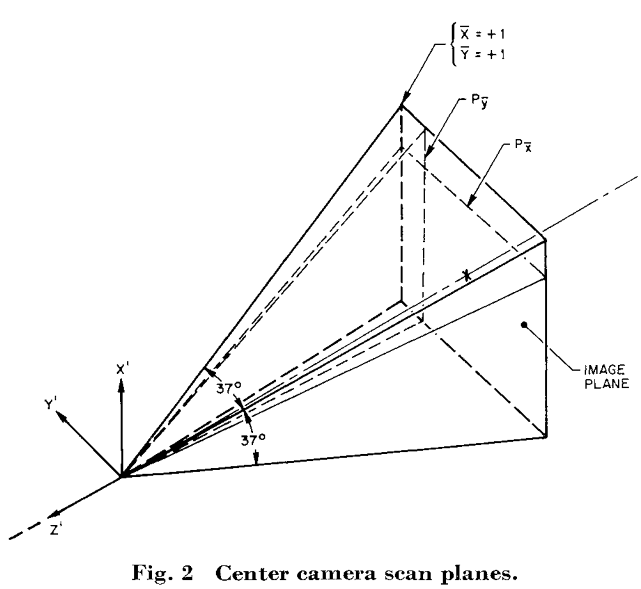


**Fig. 2 Center camera scan planes.**

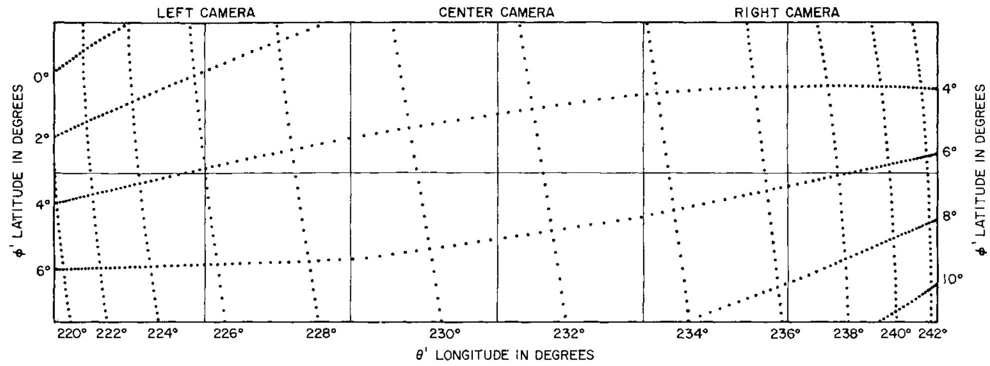


**Fig. 3 Typical result of gridding calculations for zero pitch, roll, and yaw angles.**

with the pitch axis:

$$e_P = -\sin i \sin\Omega e_X + \sin i \cos\Omega e_Y - \cos i e_Z \qquad (4)$$

Since the yaw axis is in the orbit plane, and is directed toward the center of the earth, $e_W$ is expressed as

$$e_W = -\frac{X_P}{D} e_X - \frac{Y_P}{D} e_Y - \frac{Z_P}{D} e_Z \qquad (5)$$

A unit vector along the roll axis $e_R$ that establishes a right-handed local reference frame is determined from the cross product of $e_P$ and $e_W$. With $e_R$, $e_P$, and $e_W$ expressed in terms of $e_X$, $e_Y$, and $e_Z$, the transformation from local to inertial axes is established.

If the transformation in Eqs. (2) is denoted by $A$, and the local to inertial axes transformation by $B$, the desired relationship between the body axes and the inertial axes is:

$$\begin{bmatrix} e_{X'} \\ e_{Y'} \\ e_{Z'} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} e_X \\ e_Y \\ e_Z \end{bmatrix} \qquad (6)$$

A vector with components in the body system can be readily referred to the inertial system through the use of Eqs. (6). In terms of direction cosines, a general vector **m** referenced to the body axes is written:

$$\mathbf{m} = \lambda' e_{X'} + \mu' e_{Y'} + \nu' e_{Z'} \qquad (7)$$

On substituting for $e_{X'}$, $e_{Y'}$, and $e_{Z'}$ from (6), one obtains the transformation law for the direction cosines of a vector in the body axes to the direction cosines of a vector in inertial space. This appears as:

$$\begin{bmatrix} \lambda \\ \mu \\ \nu \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \lambda' \\ \mu' \\ \nu' \end{bmatrix} \qquad (8)$$

where the $C_{ij}$ expressions are given in the Appendix.

## Gridding Procedure

The $P_{\bar{X}}$ plane intercepts the oblate spheroid earth and defines a surface curve which relates the latitude coordinate $\phi$ to the longitude coordinate $\theta$. Similarly, the $P_{\bar{Y}}$ plane specifies another surface curve. (Note that the coordinates $\phi$ and $\theta$ are referenced to the inertial $(X, Y, Z)$ axis system.) At the intersections of the two surface curves, two points are identified in the $\phi$, $\theta$ surface coordinates that correspond to a single $\bar{X}$, $\bar{Y}$ point in the vidicon image plane. This correspondence of image points vs earth surface points is formulated below. It is assumed that the satellite's position in orbit $(X_P, Y_P, Z_P)$ is a point in both the $P_{\bar{X}}$ and $P_{\bar{Y}}$ scanning planes of a camera. In general, a normal form

$$\lambda(X - X_P) + \mu(Y - Y_P) + \nu(Z - Z_P) = 0 \qquad (9)$$

can be used to characterize planes where the $(\lambda, \mu, \nu)$ quantities are the direction cosines, relative to the inertial axes of the unit normal to the plane. Assuming the oblate spheroid shape, and a geocentric latitude $\phi$,† earth surface points can be described:

$$\begin{aligned} X &= a \cos\phi \cos\theta \\ Y &= a \cos\phi \sin\theta \\ Z &= b \sin\phi \end{aligned} \qquad (10)$$

where $a$ is the equatorial radius of the earth, and $b$ is the polar radius. Substituting (10) into (9),

$$\lambda(a \cos\phi \cos\theta - X_P) + \mu(a \cos\phi \sin\theta - Y_P) + \nu(b \sin\phi - Z_P) = 0 \qquad (11)$$

With assigned values for $\lambda$, $\mu$, $\nu$, $X_P$, $Y_P$, and $Z_P$, Eq. (11) relates $\phi$ to $\theta$ on the surface of the spheroid. Now, given values $(\bar{X}, \bar{Y})$ for a point in the image plane of a camera, the direction cosines of the planes $P_{\bar{X}}$ and $P_Y$ in the body axes can be computed according to Eqs. (1) and (1′), or by similar left and right camera relations. Next, the $C_{ij}$ parameters of the transformation (8) are computed and the $(\lambda, \mu, \nu)$ values for $P_{\bar{X}}$ and $P_{\bar{Y}}$ in the inertial system are determined. Denoting the $(\lambda, \mu, \nu)$ for the $P_{\bar{X}}$ plane by $\lambda(\bar{X})$, $\mu(\bar{X})$, $\nu(\bar{X})$, and for the $P_{\bar{Y}}$ plane by $\lambda(\bar{Y})$, $\mu(\bar{Y})$, $\nu(\bar{Y})$ the following set of two equations results:

$$\begin{aligned} &\lambda(\bar{X})[a \cos\phi \cos\theta - X_P] + \mu(\bar{X})[a \cos\phi \sin\theta - Y_P] + \nu(\bar{X})[b \sin\phi - Z_P] = 0 \\ &\lambda(\bar{Y})[a \cos\phi \cos\theta - X_P] + \mu(\bar{Y})[a \cos\phi \sin\theta - Y_P] + \nu(\bar{Y})[b \sin\phi - Z_P] = 0 \end{aligned} \qquad (12)$$

Equations (12) must be solved simultaneously for the $(\phi, \theta)$ coordinates. At this point, we have the $\phi$ and $\theta$ values on the earth model stationary in the inertial reference system. The determination of latitude $\phi'$ and longitude $\theta'$ on the rotating earth are obtained from:

$$\phi' = \phi \qquad \theta' = \theta + \omega_e t_s \qquad (13)$$

given the shutter time $t_s$ for the camera.

## Solution of the Gridding Equations

The system of equations (12) relate implicitly a point $(\bar{X}, \bar{Y})$ in the image plane of a camera to a point $(\phi, \theta)$ on the surface of the earth model. Although in most instances implicit relations are resolved by an iterative technique (e.g., Newton-Raphson), we find that the set (12) allows an explicit representation of $\sin\phi$ and $\sin\theta$ in terms of an image plane point $(\bar{X}, Y)$, and an explicit resolution of $\bar{X}$, $Y$ when a point $\phi$, $\theta$ is given.

### Given $(\bar{X}, \bar{Y})$, Find $(\phi, \theta)$

The desired explicit representations of $\phi$ and $\theta$ result from the observation that $\cos\theta$ and $\sin\theta$ can be expressed solely in terms of known quantities and $\phi$. If Eqs. (12) are rewritten

† The maximum deviation between a geocentric measure of latitude and a geodetic measure of latitude is about 0.1 deg.

into the vector-matrix format, one notes that $\cos\theta$ and $\sin\theta$ can be expressed as explicit functions of $\phi$:

$$\cos\theta = \frac{1}{\cos\phi}\left[\frac{X_P}{a} + \Gamma\left(\frac{Z_P}{a} - \frac{b}{a}\sin\phi\right)\right]$$
$$\sin\theta = \frac{1}{\cos\phi}\left[\frac{Y_P}{a} + H\left(\frac{Z_P}{a} - \frac{b}{a}\sin\phi\right)\right] \quad (14)$$

where

$$\Gamma = \frac{\nu(\bar{X})\mu(\bar{Y}) - \mu(\bar{X})\nu(\bar{Y})}{\lambda(\bar{X})\mu(\bar{Y}) - \mu(\bar{X})\lambda(\bar{Y})} \qquad \lambda(\bar{X})\mu(\bar{Y}) - \mu(\bar{X})\lambda(\bar{Y}) \neq 0$$

$$H = \frac{\lambda(\bar{X})\nu(\bar{Y}) - \nu(\bar{X})\lambda(\bar{Y})}{\lambda(\bar{X})\mu(\bar{Y}) - \mu(\bar{X})\lambda(\bar{Y})}$$

Squaring and summing the expressions in (14) one obtains a quadratic equation in $\sin\phi$ that can be readily solved, with the result

$$\sin\phi = -\frac{B}{2A} \pm \frac{1}{2A}(B^2 - 4AC)^{1/2} \quad (15)$$

with

$$A = 1 + (\Gamma^2 + H^2)b^2/a^2$$
$$B = -\frac{2b}{a}\left[\Gamma\frac{X_P}{a} + H\frac{Y_P}{a} + (\Gamma^2 + H^2)\frac{Z_P}{a}\right]$$
$$C = \left[\frac{X_P}{a} + \Gamma\frac{Z_P}{a}\right]^2 + \left[\frac{Y_P}{a} + H\frac{Z_P}{a}\right]^2 - 1$$

Two solutions for $\sin\phi$ result in (15). These solutions correspond to the intersecting scan planes meeting at points on opposite sides of the earth model. When computing, the desired solution is easily obtained by comparing the answers with the latitude $\phi_v$ of the vehicle in orbit, where $\sin\phi_v = Z_P/D$. With the appropriate value for $\phi$ determined, the equation for $\sin\theta$ in (14) can be used to compute $\theta$.

**Given ($\phi$, $\theta$), Find ($\bar{X}$, $\bar{Y}$)**

Inversely, if one is given a point ($\phi$, $\theta$) on the oblate spheroid, an explicit determination of the image plane point follows from Eqs. (12). To obtain the $\bar{X}$, $\bar{Y}$ corresponding to a given $\phi$, $\theta$ set of values for a given attitude at a point in orbit, compute

$$m_1 = \cos\phi\cos\theta - X_P/a$$
$$m_2 = \cos\phi\sin\theta - Y_P/a \quad (16)$$
$$m_3 = (b/a)\sin\phi - (Z_P/a)$$

Then rewrite (12) in terms of the direction cosines relative to the body axes:

$$\alpha_1\lambda'(\bar{X}) + \alpha_2\mu'(\bar{X}) + \alpha_3\nu'(\bar{X}) = 0 \quad (17)$$
$$\alpha_1\lambda'(Y) + \alpha_2\mu'(Y) + \alpha_3\nu'(\bar{Y}) = 0$$

where

$$\alpha_1 = C_{11}m_1 + C_{21}m_2 + C_{31}m_3$$
$$\alpha_2 = C_{12}m_1 + C_{22}m_2 + C_{32}m_3 \quad (18)$$
$$\alpha_3 = C_{13}m_1 + C_{23}m_2 + C_{33}m_3$$

and the $C_{ij}$ are given in the Appendix.

But, we have determined the primed direction cosines for the center camera as relations (1) and (1′), with the direction cosines functions of the $\bar{X}$ and the $\bar{Y}$. Substituting these relations into (17), we determine that for the center camera:

$$\bar{X} = -\alpha_1/(\alpha_3\tan 18.5°) \quad (19)$$
$$\bar{Y} = -\alpha_2/(\alpha_3\tan 18.5°)$$

and proceed similarly for the left and right cameras.

## Results

An IBM 7094 simulation of the foregoing gridding procedure was compared with a simulation of the "best" of five alternative methods. Computation time and gridding accuracy were the significant attributes desired to allow "real-time" processing of meteorological data. The method described in the paper was, on the average, two times faster than the "best" alternative procedure, which employed a spherical earth model Figure 3 illustrates a typical grid of calculated points that are merged with a video representation of cloud cover viewed by the Nimbus cameras.

## Appendix: The $C_{ij}$

$$C_{11} = -\left[\frac{Z_P\sin i\cos\Omega + Y_P\cos i}{D} + \theta_W\sin i\sin\Omega - \frac{\theta_P X_P}{D}\right]$$

$$C_{12} = \left[\frac{\theta_W(Z_P\sin i\cos\Omega + Y_P\cos i)}{D} - \sin i\sin\Omega - \frac{\theta_R X_P}{D}\right]$$

$$C_{13} = -\left[\frac{\theta_P(Z_P\sin i\cos\Omega + Y_P\cos i)}{D} - \theta_R\sin i\sin\Omega + \frac{X_P}{D}\right]$$

$$C_{21} = \left[\frac{X_P\cos i - Z_P\sin i\sin\Omega}{D} + \theta_W\sin i\cos\Omega + \frac{\theta_P Y_P}{D}\right]$$

$$C_{22} = \left[\frac{-\theta_W(X_P\cos i - Z_P\sin i\sin\Omega)}{D} + \sin i\cos\Omega - \frac{\theta_R Y_P}{D}\right]$$

$$C_{23} = \left[\frac{\theta_P(X_P\cos i - Z_P\sin i\sin\Omega)}{D} - \theta_R\sin i\cos\Omega - \frac{Y_P}{D}\right]$$

$$C_{31} = \left[\frac{Y_P\sin i\sin\Omega + X_P\sin i\cos\Omega}{D} - \theta_W\cos i + \frac{\theta_P Z_P}{D}\right]$$

$$C_{32} = -\left[\frac{\theta_W(Y_P\sin i\sin\Omega + X_P\sin i\cos\Omega)}{D} + \cos i + \frac{\theta_R Z_P}{D}\right]$$

$$C_{33} = \left[\frac{\theta_P(Y_P\sin i\sin\Omega + X_P\sin i\cos\Omega)}{D} + \theta_R\cos i - \frac{Z_P}{D}\right]$$

## Reference

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